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Multi-photon transitions between energy levels in a current-biased Josephson tunnel junction

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The escape of a small current-biased Josephson tunnel junction from the zero voltage state in the presence of weak microwave radiation is investigated experimentally at low temperatures. The measurements of the junction switching current distribution indicate the macroscopic quantum tunneling of the phase below a cross-over temperature of $T^\star \approx 280\,\mathrm{mK}$. At temperatures below $T^\star$ we observe both single-photon and *multi-photon* transitions between the junction energy levels by applying microwave radiation in the frequency range between 10 GHz and 38 GHz to the junction. These observations reflect the anharmonicity of the junction potential containing only a small number of levels.

At low temperatures and small damping the dynamics of a current-biased Josephson junction is governed by the macroscopic quantum mechanics of the superconducting phase-difference across the junction (see e.g. Refs. [1, 2] and references therein). Macroscopic quantum tunneling of the phase [3], energy level quantization [3, 4] and the effect of dissipation [5] have been studied in detail [2]. Josephson junctions are solid-state quantum devices fabricated with integrated circuit technology. Their parameters can be adjusted in a wide range and can be well controlled. Josephson junction circuits have been proposed [6, 7, 8] and recently successfully tested [9, 10, 11, 12, 13] as qubits in quantum information processing [14].

In this letter we present experimental evidence of multi-photon transitions between the ground and the first excited state in a current-biased Josephson junction. The experiments have been performed below the cross-over temperature [15], where the escape of the junction from a metastable state is dominated by quantum tunneling from the quantized energy levels. Using a high resolution measurement [16] of the switching current [17], we detect the multi-photon absorption by monitoring the decay of the junction from the zero-voltage to the finite voltage state.

In the Stewart-McCumber model [18, 19], the current biased small Josephson junction is modeled as a particle of mass m_ϕ moving in an external washboard potential $U^\phi = -E_J(\gamma\phi + \cos\phi)$, see Fig. 1, according to the equation of motion $m_\phi\ddot{\phi} + m_\phi(RC)^{-1}\dot{\phi} + \partial U^\phi/\partial\phi = 0$. Here, the phase difference ϕ across the junction represents the position of the particle. The particle mass is given by $m_\phi = C(\Phi_0/2\pi)^2$, where C is the junction capacitance and Φ_0 is the flux quantum. $E_J = \Phi_0 I_c/2\pi$ is the Josephson coupling energy with the critical current of the junction I_c determining the depth of the potential. The applied bias current I normalized as $\gamma = I/I_c$ determines the tilt of the potential and the junction resistance R causes the damping proportional to the coefficient $1/RC$.

In the absence of thermal or quantum fluctuations and for $\gamma < 1$, the junction is in the zero voltage state, corresponding to the particle being localized in the the potential well. At finite temperatures $T > 0$, the particle may escape from the well at bias currents $\gamma < 1$ by thermally activated processes [17, 20] or by quantum tunneling through the barrier [3]. The rate at which both processes occur depends on the barrier height $U_0^\phi = 2E_J\left[\sqrt{1-\gamma^2} - \gamma\arccos(\gamma)\right] \approx E_J\, 4\sqrt{2}/3\,(1-\gamma)^{3/2}$, the oscillation frequency of the particle at the bottom of the well $\omega_0^\phi = \sqrt{U''^\phi(\phi_0)/m_\phi} = \omega_p\left(1-\gamma^2\right)^{1/4}$, see Fig. 1, and the damping in the junction. Here $\omega_p = \sqrt{2\pi I_c/\Phi_0 C}$ is the Josephson plasma frequency. At temperatures below the cross-over temperature $T^\star$ [15] the quantum tunneling rate dominates the thermal activation rate. The quantization of the energy of oscillations of the phase at the bottom of the well, see Fig. 1a, has been observed both below [3] and above $T^\star$ [4].

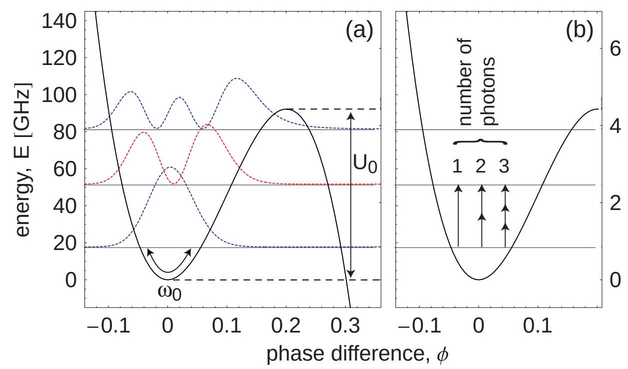

FIG. 1: a) The Josephson junction energy $U^\phi(\phi)$ calculated for the experimental parameters at the bias $\gamma = 0.995$ (solid line). Numerically calculated energy levels (dotted lines) and the squared wave functions (dashed lines) are shown. b) Multi-photon transitions between the ground state and the first excited state.

The experiments presented here were performed using a high quality $5\times5\,\mu\mathrm{m}^2$ tunnel junction fabricated on an oxidized silicon waver using a standard Nb/Al-AlO_x/Nb trilayer process. The junction had a critical current den-

sity of $j_c \approx 1.1\,\mathrm{kA/cm^2}$, a capacitance of $C \approx 1.6\,\mathrm{pF}$ and a subgap resistance of $R > 500\,\Omega$ at $T < 2.0\,\mathrm{K}$. For these sample parameters the expected energy level separation is larger than 100 GHz at zero bias. The level width is small relative to the level spacing, due to the small damping. The predicted cross-over temperature $T^\star$ is larger than 250 mK.

The sample was mounted in an rf-tight sample box on the cold finger of a dilution refrigerator. The dc-bias leads were filtered with π-type feedthrough filters at room temperature, RC-filters at the 1 K-pot of the refrigerator and thermocoax filters at the sample box in order to reduce external electromagnetic interference. A microwave signal was fed into the sample cell via a superconducting semi-rigid coaxial cable. To reduce the relative level of noise in the microwave signal, several stages of cold attenuators of a total of $-40\,\mathrm{dB}$ were used. We have verified that the power of all harmonics and subharmonics was at least 80 dB below the fundamental frequency power. For the switching current measurements [17], the current was ramped up at a constant rate of $\dot{I} = 0.245\,\mathrm{A/s}$ with a repetition rate of 500 Hz. The switching current was determined by a measurement of the time delay between the zero-crossing of the bias current and the appearance of a voltage across the junction [16].

The switching current distribution $P(I)$ of the sample in the absence of microwaves was measured in the temperature range between 4.2 K and 25 mK. These measurements indicate the thermal activation of the phase at high temperatures followed by a cross-over to quantum tunneling around $T^\star \approx 280\,\mathrm{mK}$ [16]. In this letter we present measurements performed at $T < T^\star$.

Microwaves in the frequency range between 10 GHz and 38 GHz were applied to the sample. While monitoring the $P(I)$ distributions of the junction, the microwave power $P_{\mu\mathrm{w}}$ was swept from low values, at which the $P(I)$ distribution is not changed by the microwaves, to higher values for each chosen frequency. At negligibly small microwave powers the $P(I)$ distribution is essentially determined by the unperturbed quantum tunneling of the phase from the ground state of the well. If the microwave power is increased to substantially populate the excited level, the $P(I)$ distribution becomes double-peaked. This double-peak structure smoothly varies with $P_{\mu\mathrm{w}}$, as shown for $\nu = 36.554\,\mathrm{GHz}$ in Fig. 2a. Further increasing the power, only the pronounced resonant peak is visible in the distribution. At this level of power the populations of the ground and the first excited state are equal, but the tunneling rate from the excited state is exponentially larger than that from the ground state. Thus, the $P(I)$ distribution is dominated by the resonant peak due to tunneling from the first excited state and the initial peak in the distribution disappears, see Fig. 2. Due to the resonance excitation of transitions between the two levels, the switching current distribution at this level of power is *more narrow* than in the absence of microwaves. This fact proves that the measured $P(I)$ distribution in absence of microwaves is not limited by noise in our experimental setup and that below $T^\star$ the escape indeed occurs due to quantum tunneling through the barrier.

The bias current at which the resonant peak in the $P(I)$ distribution appears depends strongly on the microwave frequency. Most strikingly, we observe resonant peaks at similar or the same bias current for very different microwave frequencies. In Figs. 2b and c, two representative density plots of the switching current distributions versus the applied microwave power are shown for the microwave frequencies 36.554 GHz and 18.399 GHz. For both frequencies the resonant peaks appear at almost identical bias currents. Both sets of data show the *pronounced narrowing of the distribution* at the resonance.

The resonant bias currents I_r, defined as the current at which the escape of the phase is maximally enhanced by the microwaves as indicated in Fig. 2 and Fig. 4, were extracted for all measured microwave frequencies ν, see Fig. 3. It is clearly observed that the resonances fall into different groups as indicated by the dashed lines. We find that ratios of the resonance frequencies ν for a fixed current I_r are given with high accuracy by ratios m/n of two small integer numbers n and m, suggesting that the observed effect is related to multi-photon transitions between energy levels of the phase.

In parabolic approximation of the potential, one expects the energy level separation in a small Josephson junction to scale with the applied bias current as $\Delta E = \hbar\omega_p(1-(I_r/I_c)^2)^{1/4}$. Therefore resonances with the external applied microwaves are expected to appear for $n\nu = \Delta E$, where n is the number of photons absorbed in the transition between two energy levels. Such multi-photon transitions between neighboring energy levels are quantum mechanically allowed [21] in the anharmonic potential for the phase of a Josephson junction due to the large diagonal matrix elements of the excited states. In Fig. 3 all data are fitted to the single formula $(1/n)\,\nu_p(1-(I_r/I_c)^2)^{1/4}$, with $\nu_p = 116\,\mathrm{GHz}$ and $I_c = 278.45\,\mu\mathrm{A}$ and $n = 1\ldots5$. The agreement between the experimental data and this simple formula for the resonance condition is excellent. The effective capacitance of the junction calculated from the fitted zero bias plasma frequency and the critical current is found to be $C = 1.61\,\mathrm{pF}$. We note that, the corresponding classical harmonic resonances, arising from the nonlinearity of the junction, have been observed experimentally [22, 23] at large applied microwave power and at high temperatures.

In Fig. 4, the escape rate $\Gamma(I)$ reconstructed from the $P(I)$ distribution is plotted for a range of microwave powers for (a) the single- and (b) the two-photon absorption processes. In both cases, at low values of $P_{\mu\mathrm{w}}$ the escape rate is a monotonic function of the bias current. With increasing $P_{\mu\mathrm{w}}$ a clear resonance develops in the escape rate. The resonant current I_r is indicated in the plot.

The enhancement $(\Gamma(P_{\mu\mathrm{w}})-\Gamma(0))/\Gamma(0)$ of the escape

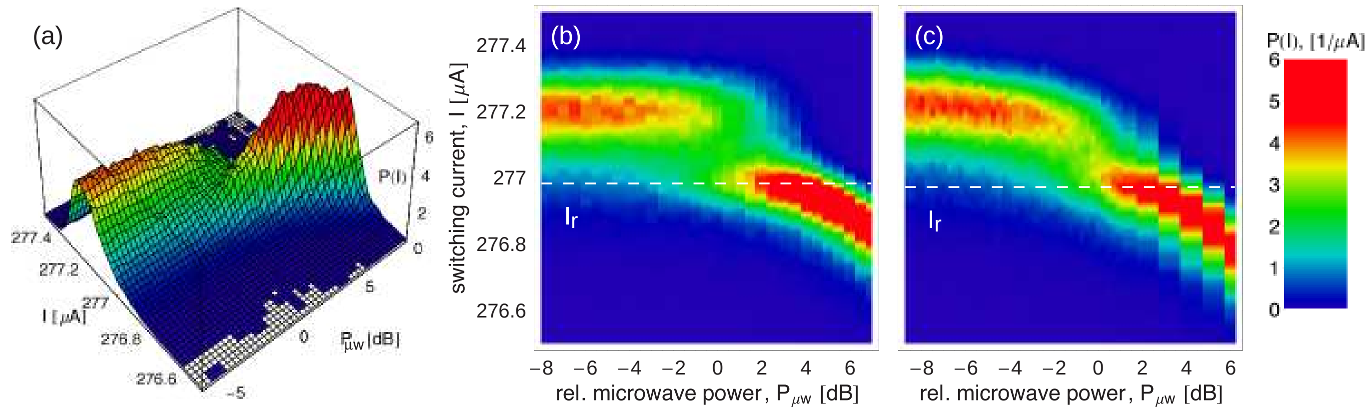

FIG. 2: a) 3D plot and b) density plot of the measured $P(I)$ distribution versus the applied microwave power $P_{\mu w}$ at $\nu = 36.554\,$GHz and $T = 100\,$mK. c) Experimental data at 18.399 GHz for the same temperature. The switching probability $P(I)$ is color coded as indicated by the scale.

rate in the presence of microwaves of power $P_{\mu w}$ is plotted for both processes in the insets of Figs. 4a and b. $P_{\mu w}$ was chosen to result in a maximum enhancement of roughly 10 in both cases. The data is fitted to a Lorentzian line shape [3] (solid line in Fig. 4). The width $\delta\nu$ of the single-photon resonance is in good agreement with the quality factor of the junction $Q \approx \nu/\delta\nu = 380$ determined from independent measurements [16]. For the two-photon process it is observed that the linewidth is approximately a factor of two larger than for the single photon process. For both processes it is observed that the line width of the transition is independent of the microwave power. This indicates that it is entirely limited by the life time of the excited state and that a coherent broadening of the resonance was not observable in these measurements. The enhancement of the escape rate due to the microwave radiation increases approximately linearly with $P_{\mu w}$ for the single photon process, whereas it increases roughly as $P_{\mu w}^2$ for the two-photon process.

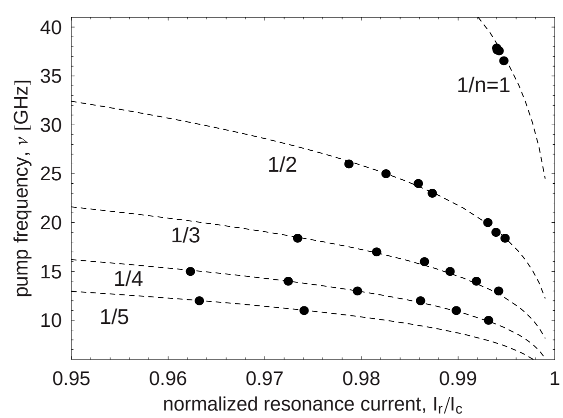

FIG. 3: Applied microwave frequency ν versus normalized resonant bias current I_r/I_c (dots). Dashed lines are a fit of the data to $\nu = (1/n)\,\nu_p(1-(I_r/I_c)^2)^{1/4}$.

We have compared our experimental results with the predictions of the Larkin-Ovchinnikov theory [24]. The bias-current dependent escape rate $\Gamma(I)$ due to tunneling from all possible energy levels was calculated using a master equation approach, considering the occupation of the energy levels in the presence of microwaves at $T = 100\,$mK. The energy levels and matrix elements were determined using the approximations introduced in Refs. [24, 25, 26], which we found to be consistent with our direct numerical solutions of the Schrödinger equation for this problem. For single-photon processes the microwave-induced transition rates between nearest-

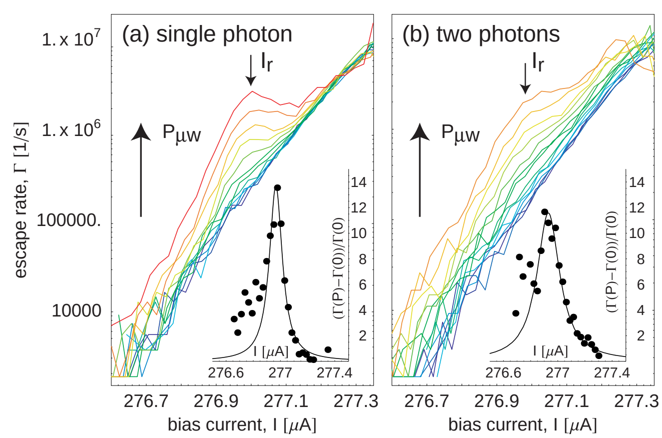

FIG. 4: Experimental escape rate $\Gamma(I)$ for (a) single-photon and (b) two-photon absorption. Different curves correspond to $P_{\mu w}$ being increased (see arrow) from zero to a value at which the maximum enhancement $\Gamma(P_{\mu w} - \Gamma(0))/\Gamma(0)$ is approximately 10. The resonance current I_r is indicated by an arrow. The insets show the enhancement of the escape rate $(\Gamma(P_{\mu w}) - \Gamma(0))/\Gamma(0)$ at the largest displayed value of $P_{\mu w}$. Symbols are data, solid lines are fits to a Lorentzian line shape.

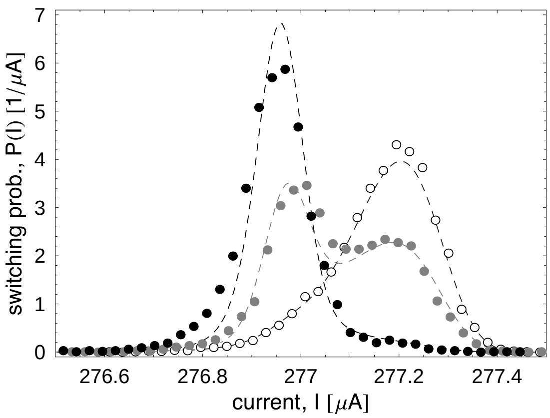

FIG. 5: $P(I)$ distributions in the presence of microwave radiation at $\nu = 36.554\,\mathrm{GHz}$ for $P_{\mu\mathrm{w}} = -7.5\,\mathrm{dB}$ (open points), 1 dB (gray points) and 4 dB (black points) measured at 100 mK. Dashed curves are calculated according to Larkin-Ovchinnikov theory.

neighbor levels j are given by [24, 25, 26]

$$W^{\mu\mathrm{w}}_{j,j+1} \propto P_{\mu\mathrm{w}}\Gamma_j \left[(2\pi\nu - \hbar^{-1}\Delta E_{j,j+1})^2 + \Gamma_j^2/4\right]^{-1}, \quad (1)$$

where Γ_j is the inverse "lifetime" of the $j+1 \rightarrow j$ transition.

In Fig. 5, the measured switching current distributions for the single photon absorption are fitted to the calculated distributions at different microwave powers for a junction capacitance of $C = 1.27\,\mathrm{pF}$, a critical current of $I_c = 278.25\,\mu\mathrm{A}$ and an effective resistance of $R = 500\,\Omega$ The capacitance obtained in this fit is about 20 % lower than that obtained from the plasma frequency. We suppose this discrepancy to be due to a microwave-induced level shift which is to be expected due to the large diagonal matrix elements of the excited states. The theory well explains the power dependence of $P(I)$, strengthening the claim that the resonances are due to the microwave induced transition of the phase from the ground state to the first excited state in the well.

In the presented experiments we have found evidence for the microwave-induced multi-photon transitions between quantized energy levels of the phase in a current-biased Josephson junction. This process could be yet another possible source of decoherence in microwave driven superconducting qubits. It could also be used for manipulating the quantum state of a qubit.

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